SECTION II.—GENERAL METEOROLOGY.

LAND AND SEA BREEZES.[1]

[Communicated to the International Meteorological Congress at Chicago, August, 1893.]

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[Dated, Harvard University, July 1, 1893.]

HISTORY OF THE THEORY.

It is the purpose of this paper to present a brief review of the most important publications on land and sea breezes, in order that it may be seen just where we stand in our knowledge of the phenomena and mechanism of these winds. The aim of the essay is, therefore, not to present any new facts, but simply to summarize those facts which have been already brought out by preceding writers. In compiling this review the present writer has merely condensed what is contained in the report on the sea breeze published by Harvard College Observatory (1). That report contains an historical account of the theory of land and sea breezes, as well as the results derived from the study of the sea breeze undertaken by the New England Meteorological Society during the summer of 1887.

One of the earliest as well as one of the best descriptions of land and sea breezes is given by the old navigator Dampier (2). The account is so good that it deserves quotation at some length here.

Sea breezes, he writes, generally speaking, are no other than the common trade wind of the coasts on which they blow, with this difference, that whereas all trade winds, whether they are those that I call the general trade winds at sea, or coasting trade winds * * * do blow as well by night as by day, with an equal briskness, except when tornadoes happen; so contrarily sea winds are only in the day and cease in the night; and as all trade winds blow constantly near to one point of the compass * * * on the contrary these sea winds do differ from them in this, that in the morning, when they first spring up, they blow commonly as the trade winds on the coast do, at or near the same point of compass; but about mid-day they fly off about two, three, or four points farther from the land, and so blow almost right in on the coast, especially in fair weather, for then the sea breezes are truest. * * * These sea breezes do commonly rise in the morning about nine o'clock, sometimes sooner, sometimes later; they first approach the shore, so gently, as if they were afraid to come near it, and ofttimes they make some faint breathings and, as if not willing to offend, they make a halt, and seem ready to retire. * * * It comes in a fine, small, black curle upon the water, when as all the sea between it and the shore not yet reach'd by it is as smooth and even as glass in comparison; in half an hour's time after it has reached the shore it fans pretty briskly, and so increaseth gradually till twelve a-clock, then it is commonly strongest, and lasts so till two or three a very brisk gale; about twelve at noon it also veers off to sea two or three points, or more, in very fair weather. After three a-clock it begins to die away again, and gradually withdraws its force till all is spent, and about five a-clock, sooner or later, according as the weather is, it is lull'd asleep and comes no more till the next morning. * * *

Land breezes * * * are quite contrary to the sea-breezes; for those blow right from the shore, but the sea breeze right in upon the shore; and as sea breezes do blow in the day and rest in the night; so on the contrary, these do blow in the night and rest in the day, and so they do alternately succeed each other. For when the sea breezes have performed their offices of the day, by breathing on their respective coasts, they in the evening do either withdraw from the coast, or lie down to rest; then the land winds, whose office is to breathe in the night, moved by the same order of divine impulse, do rouse out of their private recesses and gently fan the air till the next morning; and then their task ends and they leave the stage. * * *

The sea breezes indeed are very comfortable and refreshing; for the hottest time in all the day is about 9, 10, or 11 o'clock in the morning, in the interval between both breezes; for then it is commonly calm, and then people pant for breath, especially if it is late before the sea breeze comes, but afterwards the sea breeze allays the heat.

The occurrence of the land and sea breezes, which has been observed and described by very many writers, although by none more clearly than by Dampier, was early rightly explained as being due to the diurnal warming and cooling of land more than water. A common form of statement of this theory, which we may call the simple convectional theory, is found in Buchan's (3) Text Book of Meteorology (1871), in which the author says that the land is

heated to a much greater degree than the sea during the day, by which the air resting on it being also heated, ascends, and the cooler air of the sea breeze flows in to supply its place. But during the night the temperature of the land and the air above it falls below that of the sea, and the air thus becoming heavier and denser flows over the sea as a land breeze.

This simple convectional theory has been almost universally accepted by those who have written on this subject. Among the more prominent authors who have adopted this theory may be mentioned the following:

Capper, 1801.	Fitzroy, 1863.
Maury, 1856.	Marié-Davy, 1879.
Mühry, 1860.	Geikie, 1879.
Herschel, 1861.	Loomis, 1885.
Dove, 1862.	Mohn, 1887.

This theory requires that the sea breeze shall begin near the shore and gradually extend backwards out to sea, for it is the air near the shore which will first flow in to take the place of the warmed and ascending air over the land. Maury in his "Physical Geography of the Sea," describes this action as follows:

About 10 in the morning the heat of the sun has played upon the land with sufficient intensity to raise its temperature above that of the water. A portion of this heat being imparted to the superincumbent air, causes it to rise when the air, first from the beach, then from the sea, to a distance of several miles, begins to blow in.

Mühry (4) also says: "The sea breeze begins in the neighborhood of the coast, and gradually prolongs itself backwards, because its motive is 'aspiration' in the region in front of it." The sea breeze, however, does not begin at the shore, but first appears some miles out at sea, gradually working its way towards the shore. This fact was clearly brought out in Dampier's account, already quoted, his statement being that the breeze comes as "a fine, small black curle upon the water, when as all the sea between it and shore not yet reached by it is as smooth and even as glass in comparison." Jansen, quoted by Maury, says that:

Far out upon the sea arises and disappears alternately a darker tint upon the otherwise shining sea-carpet; finally, that tint remains and approaches; that is the long-wished-for sea breeze; yet it is sometimes one, yes, even two hours before that darker tint is permanent, before the sea breeze has regularly set in. * * * We welcome the first breath from the sea; it is cooling, but it soon ceases; presently it is succeeded by other grateful puffs of air, which continue longer; presently they settle down into the regular sea breeze with its cool and refreshing breath.

This off-shore beginning of the sea breeze has also been carefully described by several other observers, including

[1] The present paper was prepared for publication in 1901, but has been delayed for the reasons stated in the REVIEW for February, 1914, 42: 93.

some of those who took observations for the New England Meteorological Society in 1887, and there is no reason to doubt that this feature is a general characteristic of this wind, and as such it should be accounted for by the theory. As the simple convectional theory, above outlined, stands, it gives no explanation of this fact.

This failure of the generally accepted theory to explain the off-shore beginning of the land breeze led Laughton (5) to put forward a different theory. He does not consider the differences of temperature between the land and sea sufficient to produce land and sea breezes, and finding that the sea breeze is most distinct on coasts with a mountain range in the background, gives the following explanation. The sea breeze is not due to a decrease of pressure in front, over the land, but to an increase of pressure behind, over the sea, this increase of pressure being derived from the water vapor formed over the sea, aided, in the case of mountainous coasts, by the withdrawal of water in the rainfall there induced. The land breeze is ascribed to the accumulation aloft of the air brought in by the sea breeze, which descends in the evening and slides back towards the sea again. Admitting that vapor pressure may aid in producing the sea breeze, and that the presence of mountains near the coasts may also be of help in confirming the circulation, still it is impossible to believe that the air brought in by the sea breeze can remain stationary aloft near the coast until the cessation of the on-shore wind allows this mass of air to move seaward again.

Another view, which has been adopted by Hann and Scott, is that of Blanford, whose recent death [Jan. 23, 1893] has removed from the ranks of meteorologists one of the most active and valuable of workers. Blanford has shown that the air over the land, heated by contact with the warm ground, is expanded much more than that over the sea, where the heat is largely used in evaporating the water. The result is that the isobaric surfaces are expanded over the land, and given a slope seaward, the steeper slope being aloft. Down this slope the upper strata of air over the land slide seaward, and thus an increment of pressure is produced at some distance off-shore. This results in an inflow of air from over the sea towards the land, where there is rarer atmosphere and lower pressure, and in this way the off-shore beginning of the sea breeze is explained. During the night the opposite conditions prevail, and the land breeze results.

Köppen (6) has objected to Blanford's theory on the ground that the removal of the upper air from over the land in the morning will relieve the pressure over the land by just as much as the pressure over the sea is increased, and that therefore the air nearest the shore will flow in first, as in the case of the draught of a fire. He therefore suggests as the cause of the off-shore beginning of the sea breeze that the air near the land is held back by friction with the ground, thus forming a cushion against which the sea breeze strikes and by which it is kept out from the shore until the differences of pressure have become great enough to overcome the opposition. Although this friction may be important in the case of rugged and mountainous coasts, it hardly seems possible that it can be efficacious on low and sandy shores.

The difficulty of giving an adequate explanation of the offshore beginnings of the sea breeze has been successfully met by Seemann (7), a captain in the German merchant service. After careful observation of the phenomena of sea breeze action in different harbors to which his occupation as shipmaster has taken him, and especially in the harbor of Valparaiso, where the advance of the breeze toward the shore is very clearly marked, Seemann puts forward a new view, which is as follows: As the land is warmed in the morning more than the sea, and the air over the land more than that over the sea, there comes to be a broad mass of expanding air over the land, pushing not only upward against the cooler overlying air, but outward as well. It is this lateral expansion of the warmed air over the land which prevents the early and immediate entrance of the sea breeze, and which often produces an increase of the off-shore land breeze in the early morning hours. This latter point is well shown at Rio Janeiro, where the land breeze is strongest about 8 a. m. and is of great service in helping vessels to leave that port, and also at many other places. The sea breeze is kept out because the warming air over the land is able by its increasing expansive force, to balance the increasing weight of the air over the sea. As long as the temperature over the land rises rapidly the expansive force of the warming air is able to keep out the sea breeze. But at 11 a. m., or thereabouts, when the increase of temperature over the land is no longer so rapid and when the increase of pressure over the sea has become greater, the sea breeze can push forward nearer the shore. Finally, when there is no longer an increase of temperature over the land, there is no more opposition, and the sea breeze can blow all the way to the shore and beyond it for some miles inland.[2]

A series of tables, given by Blanford, showing the changes of pressure over the land at Calcutta and over the sea at the Sand Heads Lightship, 60 miles offshore, give a striking confirmation of Seemann's theory. From these tables it is seen that between 1 and 2 a. m. the pressure over the land becomes greater than that over the sea, and then the land breeze begins. This difference in pressure increases till 4 a. m., and then slowly decreases till 6 a. m. A rise in pressure is then noted, owing to the expansion of the air over the land by warming, the maximum being reached at 9 a. m. After this the difference in pressure becomes less as the temperature rises more slowly and the upper layers of air flow off seaward. At 1 p. m. the conditions change and the sea breeze comes in, reaching its greatest velocity between 3 and 6 p. m., when the differences in pressure are greatest. By 8 p. m. the difference in pressure is small; by 9 p. m it is almost gone, and at 1 a. m. the conditions are reversed again.

MAIN FEATURES OF LAND AND SEA BREEZES.

Having thus briefly reviewed the history of the theory of land and sea breezes, it remains for us to note some of the main features of these winds. These we shall take up under the headings: (1) Relations of land and sea breezes to general weather conditions; (2) Dimensions and physical features of the sea breeze; (3) Effects of the earth's rotation on the course of the breezes; (4) Combination of land and sea breezes with other winds; (5) Effect of land and sea breezes on the distribution of aqueous vapor. We must of necessity treat each of these headings very briefly.

Relations to general weather conditions.—In most of the accounts of land and sea breezes mention is made of the fact that they are best developed in fine summer weather,

[2] A more recent discussion of the circulation in the sea breeze, and its origin, is given by Sandström in Bull., Mt. Weather obsy., 1912, 5, p. 90-91.—EDITOR.

and of their absence on cloudy or stormy days. Dampier, with his keen observation, noted this in his account which we have quoted, for he speaks of the sea breeze as being "truest" in fair weather. The reason for this is obvious. On clear days, when the anticyclonic conditions and faint gradients generally prevail, the conditions for the diurnal warming and cooling necessary for the production of these winds are usually favorable, and there is little or no interference through the presence of strong cyclonic winds. In the torrid zone, where even conditions prevail throughout the year, these breezes are a persistent climatic feature. In the temperate zones they are characteristic seashore phenomena of fine summer weather, when the conditions are favorable.

Dimensions and physical features.—The most reliable observations on the vertical thickness or height of the sea breeze were made by C. T. Sherman at Coney Island, near New York, some few years ago by means of captive balloons. From 12 observations thus made the average height of the sea breeze was found to be 410 feet, and the base of the return current was noted at 525 feet. The average temperatures were found to be 77°.1 at the earth's surface; 75° at 100 feet above the surface and 74°.5 at 200 feet above the surface. Balloon ascents by Duruof and Tissandier at Calais, August 16, 1868, and by Eloy and Lhoste at Boulogne, June 6, 1883, furnish good evidence as to the lower on-shore breeze and the upper return current, but give no reliable measurements.

As regards the distance of penetration inland, and the rate at which the sea breeze advances, Blanford states that the breeze is sometimes felt at Calcutta, 68 miles from the sea. In New Jersey it has been noted at a distance of 40 miles from the shore, the time taken by the breeze in advancing over that distance being four or five hours. In New England the inland limit of the sea breeze is between 10 and 20 miles and the rate of advance at the shore is 10 to 15 miles an hour, but diminishes rapidly further inland. The breezes are usually of very moderate velocity, unless combined with other winds, the velocity of the land breeze being less than that of the sea breeze.

The effect of the on-shore breeze during the day is naturally to lower the temperature of the places reached by it, the result being that the maximum temperature of the day occurs in the middle of the morning instead of in the early afternoon. The sea breeze, with its cooling effects and its fresh sea smell, is one of the most delightful features of the climate of many seashore districts, adding much to the comfort as well as to the health. The breeze is often noted as causing the sky to darken a little toward the horizon as it approaches. This fact is in all probability due to the greater cleanness of the air from the sea, the luminosity of the sky depending as is well known, on the amount of dust or haze in it.

Effects of the earth's rotation.—The deflective effect of the earth's rotation, which tends to turn moving bodies to the right of a direct course in the Northern Hemisphere and to the left in the Southern, is well established in the case of the land and sea breezes. The veering in this hemisphere of these breezes has long been recognized. Capper in 1801, noted the fact that the on-shore wind seemed regularly to follow the course of the sun, passing around every point of the compass in 24 hours. Chambers, in his discussion of the hourly wind observations at Bombay **(8)**, has brought out this point very clearly. The sea breeze when it is best developed, during the off-shore winter monsoon, begins in the west and veers to the north. A similar example is found in H. A. Hazen's (9) "Report on Wind Velocities at the Lake Crib and at Chicago," in which it is seen that the mean hourly direction of the wind at Chicago during July, 1882, changed from almost due east at 1 p. m. through southeast to almost south at 10 p. m. In the investigation of the sea breeze in New England during the summer of 1887 this veering was also very distinctly recognized in several cases. The veering (in this hemisphere) is noted in the afternoon and at night, as the sea and land breezes are derived from a greater and greater distance from the shore.

Combination with other winds.—The combinations of the land and sea breezes with the other winds which may prevail during their occurrence, are many. On mountainous tropical islands the sea breeze combines with the diurnal valley wind blowing up the mountain sides to form clouds and often rain on the mountains. As soon as the land breeze begins the clouds disappear. Hann, in his "Klimatologie" **(10)**, notes that where the sea breeze has the same direction as the prevailing wind it usually increases during the afternoon to a gale, while the land breeze is hardly noticeable. On coasts with offshore winds the land breeze is stronger, while the sea breeze frequently simply lessens the velocity of the prevailing wind. Koeppen **(11)** maintains that in cases where the prevailing winds are offshore the sea breeze either overcomes these winds entirely and is felt as a distinct wind from the sea, or else the trades continue uninterruptedly throughout the day. Other examples of the combination of land and sea breezes with prevailing winds are given by Maury, Loomis, Schmid, and others.

Effect on the distribution of aqueous vapor.—In his report on atmospheric circulation, based upon the results of the *Challenger* Expedition, Buchan points out that land and sea breezes have a marked disturbing effect on the diurnal distribution of aqueous vapor in the lower stratum of the atmosphere. Over the open sea, at some distance from land, the typical curve of humidity has its maximum and minimum at the times when the temperature of the sea surface and of the air lying over it has its maximum and minimum. Under the influence of the land breeze the minimum humidity is not reached till about 6 a. m., and there is further a secondary minimum of humidity for some hours between 10 a. m. and 4 p. m., during the time when the ascending current from the land is strongest and the sea breeze is also strongest. This diminution in the amount of aqueous vapor near the land Buchan believes to be due to an intermixture with the sea breeze, of descending currents of air coming down to take the place of the air which has been removed by the ascending currents over the land. In this way the dryness of the air of the sea breeze is explained, this dryness indicating a descent from aloft rather than an inflow for some distance over the sea.

The only systematic study of the sea breeze yet made, so far as known to the writer, was that undertaken by the New England Meteorological Society in the summer of 1887. The results derived from that one season's work, interesting and valuable in themselves, serve to show how much more might be accomplished by similar investigations elsewhere. There remain many points which need further study. Simple observations of the height of the land and sea breezes and of the return upper currents by means of captive balloons, and barometric records from stations a few miles inland, on the shore and on islands offshore, would afford excellent tests of Seemann's theory. Such observations could be very easily made and the work is certainly most attractive.

Although in considering land and sea breezes, we are dealing with one of the less important meteorological phenomena, it is well, in closing this review, to call attention to the correlation of these breezes with other winds of more importance but of similar origin. The warming and cooling of the land which gives rise to the on- and offshore breezes we have been studying, is a diurnal affair depending on the warming by the sun's heat by day and the cooling by radiation and conduction by night. The changes of temperature and of pressure to which these breezes are due, are not, however, restricted to the coastal region, as the breezes themselves are, but extend all over the land surface. Now what we see as a diurnal phenomenon in the case of the land and sea breezes, we see as a seasonal phenomenon in the increase of pressure over the continents in winter and the decrease of pressure in summer, whereby they become alternately areas of high and low pressure and their wind circulation changes accordingly. A winter continent, therefore, has outflowing winds, and a summer continent has inflowing winds, and these, which may be called continental winds, combine with the larger class of terrestrial winds to form the general winds of the earth. The class of seasonal, or continental winds is simply a larger example of the smaller class of land and sea breezes. In the former case the continuance of the temperature and pressure conditions is for some months at a time; in the latter it is for a few hours only. In the former case, therefore, a general continental circulation of the winds can be established; in the latter there is only time for the establishment of a local and incomplete circulation.

NOTE.—ADDED JULY 24, 1893.

Since preparing the above review, the writer has received a copy of Dr. Otto Krümmel's "Geophysikalische Beobachtungen der Plankton-Expedition" (Kiel & Leipzig, 1893), in which are presented the results derived from the meteorological observations made by Dr. Krümmel during the scientific exploring voyage of the Plankton Expedition in the Atlantic Ocean during July–November, 1889. Dr. Krümmel calls attention to a point in connection with sea breezes which is worthy of note here, and of careful observation in any future investigation of this class of winds. He noticed that during the time the vessel *National* was anchored off Para the sea breeze was most marked during a flood tide. This fact the author finds referred to in several previous accounts of land and sea breezes. In Staff Commander James Penn's "Sailing Directions for the West Coasts of France, Spain and Portugal" (London, 1867, p. 273) it is stated that at Cadiz—

> The sea breezes vary from west to north-northwest and are generally strongest at the full and change of the moon, when they not unfrequently blow during the whole night. They set in most commonly with the flood * * *.

Further, in the "Annalen der Hydrographie," 1887, p. 164, the captain of the German cruiser *Habicht* states that at Kamerun the sea breeze is the strongest when the flood tide comes in the afternoon. The explanation of this fact is found in the mechanical raising, by the rising tide, of the mass of air lying over the water near the shore, thus causing stronger gradients aloft and consequently a more active circulation.

REFERENCES.

(1) **Davis, W. M., L. G. Schultz, & Ward, R. DeC.** An Investigation of the sea breeze. Annals, Astron. obsy. Harv. Coll., **21**, pt. 2, Cambridge, Mass. 18:0. p. 215–263. f°.

(2) **Dampier, William.** Voyages and descriptions. London, 1705. *See also* Dampier's voyages ... edited by John Masefield. 2 v. E. P. Dutton & Co., New York, 1906. 8°. v. **2**, pt. 3, chapt. 4, pp. 251–264.

(3) **Buchan, A.** Introductory textbook of Meteorology. Edinburgh, &c. 1871. p. 132, § 334.

(4) **Mühry.** Über die Theorie der Land- und See-Winde. Ztschr. d. Oesterr. Ges. f. Meteorol., 1868, **3**, p. 475.

(5) **Laughton.** Physical geography in its relation to the prevailing winds. 1873. 400 p. 8°.

(6) **Köppen, W.** *In* Meteorol. Ztschr., 1884, **1.** Jhrg., p. 40.

(7) **Seemann.** Über Land- und See-winde und deren Verlauf. Wetter, Berlin, 1884, p. 79, 101, 124.

(8) **Chambers.** Meteorology of the Bombay Presidency. 1879. p. 109.

(9) **Hazen, Henry A.** Report on wind velocities at the Lake crib and at Chicago. Washington. 1883. (U. S. Signal service notes, No. VI.)

(10) **Hann, Julius.** Handbuch der Klimatologie. 3te Aufl. Stuttgart. 1908–1911. 3 v. 8°. v. **1**, p. 151.

(Same) translated by Ward. New York. 1903. p. 155.

(11) **Köppen, W.** *In* Ztschr. Oesterr. Gesellsch. f. Met., Wien, 879, **11**, p. 377.

GRAPHICAL INTEGRATION OF FUNCTIONS OF A COMPLEX VARIABLE WITH APPLICATIONS.

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[Dated, University of Alberta, November 5, 1913.]

Many problems in mathematics can be solved more simply by graphical than by analytical methods; especially is this true when the problem is presented graphically, and we wish a graphical representation of the solution. The object of this present paper is to give some of the results of graphical integration of functions of a complex variable as obtained by the author and published in part in his Dissertation (1).

Functions of a complex variable arise in a great many problems of mechanics and physics. In these the graphical method of solution is of great advantage, and gives results in which the errors of graphic methods are so small that they may be disregarded.

As an introduction to the integration of functions of a complex variable I wish to give a short and accurate method of graphically integrating functions of a real variable (2), since graphical integration of functions of a complex variable can be reduced to repeated successive graphical integrations of functions of a real variable.

In this paper I shall only consider the mathematical side of the problem and give one or two examples worked out in detail, so that the physicist or student of applied mechanics can easily apply my method to other problems.

1. GRAPHICAL INTEGRATION OF FUNCTIONS OF A REAL VARIABLE.

A function $f(x)$ of the real variable x can be represented graphically in the x,y plane. On the x axis (fig. 1) we take a number of points x_1, x_2, x_3, $\cdots$; through them draw the ordinates cutting the curve $f(x)$ in the points a, A, B, C, $\cdots$. We now draw M_1c parallel to the x axis so that the area of aM_1b equals the area of bcA. In the same way we draw de; fg; hi; $\cdots$ and produce them to cut the y axis in the points M_1, M_2, M_3, M_4, $\cdots$. Take a point P on the x axis so that Px_1 shall equal unity; and join P to these points M_1, M_2, M_3 $\cdots$. Now the